**MATH 3100 – Learning objectives to meet for the final exam**

The final exam is cumulative and all of the material discussed during the semester is fair game. **The following review sheet covers only material after Exam #3.** You are strongly encouraged to consult earlier review sheets and past exams in your preparation for the final.

## What to be able to state (since last exam)

### Basic definitions

You should be able to give complete and precise definitions of each of the following items:

- Taylor series of $f$
- $n$th Taylor polynomial $P_n^f(x)$

### Big theorems

Be familiar with all of the following theorems, making sure that you can give full statements for those not already spelled out below. For results proved in class, describe the components and main ideas of the proof.

- If $f$ is represented by a power series (centered at 0) on some interval $(-r, r)$, where $r > 0$, that power series is the Taylor series of $f$.
- Taylor's theorem: Let $n$ be a nonnegative integer. Suppose $f$ is $(n+1)$-times differentiable on an open interval $I$ containing 0. For every nonzero $x \in I$,

$$f(x) - P_n^f(x) = \frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1} \quad \text{for some } c \text{ strictly between 0 and } x.$$

## What to expect on the final exam

The format of the final exam will be similar to your three midterms but the length will be approximately double: You should expect the number of questions to land in the closed interval $[8, 10]$. **Among other things**, there will be ...

- A problem asking you to establish the value of a limit directly from the definition of a limit.
- A problem asking you to determine convergence/divergence of given series (possibly also absolute or conditional convergence).
- A problem asking you to apply Taylor's theorem.

# Practice problems for Taylor series/polynomials

1. Let $f(x) = 3x/(1+2x^5)$.

   (a) Find a power series that represents $f(x)$ on $(-r, r)$ for a value of $r > 0$.

   (b) Find the Taylor series of $f(x)$. (Hint: You can do this without computing derivatives of $f$ !)

   (c) Find the exact value of $f^{(4)}(0)$. Do the same for $f^{(6)}(0)$. Justify your answers.

2. (a) Let $f(x) = (1+x)^{1/3}$. Find $P_3^f(x)$.

   (b) Show that

$$|\sqrt[3]{2} - P_3^f(1)| < \frac{80}{81 \cdot 24}.$$

   (c) Is $P_3^f(1)$ larger or smaller than $\sqrt[3]{2}$?

   Justify your answers in (b) and (c) using Taylor's theorem.

3. Let $f(x) = e^x$ and $P(x) = 1 + x + x^2/2 + x^3/6$.

   Using Taylor's theorem, prove the following claim:

   The graph of $f(x)$ is entirely above (or touching) the graph of $P(x)$. In other words, for every real number $x$,

$$e^x \geq P(x).$$

   **Be sure that your argument works for all $x$, including negative values of $x$.** As illustration, the graphs of $f(x)$ (dotted) and $P(x)$ (dashed), from $x = -2$ to $x = 2$, are shown below.