MATH 3100 – Homework #5

posted October 8, 2025; due October 15, 2025

There you stand, lost in the infinite series of the sea... – Herman Melville

Section and exercise numbers correspond to the online notes. Assignments are expected to be **neat** and **stapled**. **Illegible work may not be marked.**

Problems required for everyone

1. Prove that the series $\sum_{k=1}^{\infty} \frac{k}{k+1}$ diverges.

   Hint: You can do this in one line by citing an appropriate result from class.

2. §2.1: 13

3. §2.1: 15

4. §2.2: 1(a,c,d,f,h,j,l)

   Hint: None of these parts require the integral test!

5. §2.2: 2

   Hint: First prove that $a_n^2 \leq a_n$ eventually. Then finish the problem using the Eventual Comparison Test.

6. §2.2: 3

7. §2.3: 3(b,e,f,g)

8. §2.3: 9

Recommended problems (NOT to turn in)

§2.1: 1, 2, 3, 4, 5, 6, 8, 10, 14
§2.2: 1(b,e,g,i,k)

MATH 3100H problem

9. Let $f(x)$ be a function that is nonnegative and decreasing for $x \geq 1$. In this case, we know from class that

$$f(n+1) \leq \int_n^{n+1} f(t)\,\mathrm{d}t \leq f(n)$$

for every natural number n. Define a sequence $\{\gamma_n\}$ by setting

$$\gamma_n = f(1) + f(2) + \cdots + f(n) - \int_1^{n+1} f(t)\,\mathrm{d}t.$$

   (a) Show that $\{\gamma_n\}$ is an increasing sequence.

   (b) Prove that $\gamma_n \leq f(1) - f(n+1)$ for each $n \in \mathbf{N}$.

   (c) Show that $\{\gamma_n\}$ converges to a real number γ satisfying $0 \leq \gamma \leq f(1)$.

Remark. The special case when $f(x) = 1/x$ has a distinguished history. Since $\int_1^{n+1} \frac{\mathrm{d}t}{t} = \ln(n+1)$, for this function $f(x)$ we have that

$$\gamma_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n} - \ln(n+1).$$

The corresponding limit γ is known as the Euler–Mascheroni constant, and it can be shown that $\gamma = .57721\ldots$. The upshot: For large numbers n, the sum $1 + \frac{1}{2} + \cdots + \frac{1}{n}$ is closely approximated by $\ln(n+1) + 0.57721$.