MATH 3100 – Homework #2

posted January 22, 2024; due by end-of-day on January 31, 2024

'Obvious' is the most dangerous word in mathematics. — E.T. Bell

Section and exercise numbers correspond to the online notes. Assignments are expected to be **neat** and **stapled**. **Illegible work may not be marked.**

Required problems

1. Let $a_n = 2n$ for all $n \in \mathbf{N}$, and let $b_n = 2^n$ for all $n \in \mathbf{N}$. Show that $\{b_n\}$ is a subsequence of $\{a_n\}$ by writing down a strictly increasing function $g\colon \mathbf{N} \to \mathbf{N}$ with $b_n = a_{g(n)}$ for all $n \in \mathbf{N}$.
2. §1.3, Exercise 8 (no proofs necessary)
3. §1.3, Exercise 9 (no proofs necessary)
4. §1.3, Exercise 13
5. §1.3, Exercise 15
6. Let $\{a_n\}$ be the sequence defined by $a_n = n^2 - 7n + 11$ for all $n \in \mathbf{N}$. Find a value of $N \in \mathbf{N}$ such that $a_n \leq a_{n+1}$ for all $n \geq N$. Justify your answer.
7. Show that if $\{a_n\}$ is a sequence, then $\lim_{n\to\infty} a_n = 0$ if and only if $\lim_{n\to\infty} |a_n| = 0$. (Remember that an if-and-only-if statement requires a proof for **both** directions.)
8. In class, we will show that if $0 < r < 1$, then $\lim_{n\to\infty} r^n = 0$. We will also show that if $r > 1$, then $\{r^n\}$ is not bounded. You may assume these two results for this exercise.
   (a) Suppose that $-1 < r < 0$. Prove that $\lim_{n\to\infty} r^n = 0$.
   (b) Suppose that $r < -1$. Prove that $\{r^n\}$ is unbounded. Deduce that $\{r^n\}$ diverges in this case.

   Hint: Problems 5 and 7 (above) may be useful.
9. §1.4, Exercise 2
10. §1.4, Exercise 8

Recommended problems (NOT to turn in)

§1.3: 14, 17, 18, 19, 20, 25
§1.4: 1, 3, 4, 5, 6, 7