MATH 3200 – Learning objectives to meet for Exam #2

The exam will cover Chapters 4 and 5 of the course notes, through what is discussed by the end of class on Monday, 11/11. While you *are* responsible for reading the notes, the exam will not ask about definitions or concepts not covered in class or on homework.

What to be able to state

Basic definitions

Be able to give concise, complete, and precise definitions of each of the following terms.

- set containment ($A \subseteq B$)
- set equality ($A = B$)
- the empty set $\emptyset$
- $A \cup B$, $A \cap B$, $A \setminus B$
- Commutative Property, Associative Property, Distributive Property, and de Morgan's laws
- set product $A \times B$
- the power set $\mathcal{P}(S)$
- unions $\bigcup_{i \in I} A_i$ and intersections $\bigcap_{i \in I} A_i$, where I is an index set
- relation on sets S and T, relation on a set S
- domain and range of a relation
- reflexive, symmetric, and transitive
- equivalence relation
- equivalence class $[x]$
- function from S to T (only if covered by Monday, 11/11)
- domain, codomain, range, image, preimage (only if covered by Monday, 11/11)

What to be able to do (not comprehensive)

You may be called on to perform any or all of the following tasks.

- Distinguish between element containment ($\in$) and subset containment ($\subseteq$).
- Combine sets using standard operations (unions, intersections, set subtraction, products, and power sets).
- Establish containment between sets, or equality of sets, using "element chasing."

- Establish set identities using the "beyond element chasing" method. Label when you use the Commutative Property, the Associative Property, the Distributive Property, and de Morgan's laws.
- Carry out proofs involving product sets and/or power sets.
- Compute the number of elements in a product $A \times B$ of two finite sets, or the number of elements in the power set $\mathcal{P}(A)$ of a fintie set.
- Identify the domain and range of relations.
- Given a relation, determine whether it is symmetric, whether it is reflexive, and whether it is transitive.
- Be able to manipulate the notions of symmetric, reflexive, and transitive in proofs.
- Decide whether a given relation is a function.
- Determine the domain, codomain, and range of a given function. Compute images and preimages.

What to expect on the exam

You can expect 5 (possibly multi-part) questions on the exam. These will include:

- a multipart TRUE/FALSE question testing basic understanding of key concepts
- at least one problem assessing your ability to prove set containment or set equality using element chasing
- at least one problem assessing your ability to determine whether a specified relation is symmetric, reflexive, and/or transitive

Sample problems

1. Determine whether each of the following statements is `TRUE` or `FALSE`. No justification required.
   (a) $\{\emptyset\} \in \{\emptyset\}$
   (b) For every pair of sets A and B, we have $A \subseteq A \cap B$.
   (c) $R = \emptyset$ is a symmetric relation on the set $S = \{1, 2, 3\}$.
   (d) If $A = \{1, 2, 3\}$ and $B = \{\text{red}, \text{green}, \text{blue}\}$, then $|A \times B| = 6$.
   (e) Let R be the relation on $A = \{1, 2, 3\}$ and $B = \{\text{red}, \text{green}, \text{blue}\}$ defined by $R = \{(1, \text{red}), (2, \text{red}), (3, \text{red})\}$. Then R is a function from A to B.

2. (a) What do we mean when we say two sets A and B are **equal**?
   (b) Suppose A, B, C are subsets of a universal set U. Using element chasing, prove that
   $$(A \setminus B) \cap C = (A \cap C) \setminus B.$$
   No credit will be given for a proof using the "beyond element-chasing" approach.

3. (a) If X is a set, what is meant by the **powerset** of X?
   (b) Suppose A and B are subsets of a universal set U. Prove that $A \cap B = \emptyset$ if and only if $\mathcal{P}(A) \cap \mathcal{P}(B) = \{\emptyset\}$.

4. Let R be the relation on the real numbers $\mathbf{R}$ defined by
   $$x\,R\,y \quad \text{if} \quad |x - y| \leq 1.$$
   (a) Is R reflexive? Justify your answer.
   (b) Is R symmetric? Justify your answer.
   (c) Is R transitive? Justify your answer.

5. (a) Let U and V be sets. Define the product set $U \times V$.
   (b) Let A, B, and C be nonempty sets. Suppose $A \times B \subseteq B \times C$. Prove that $A \subseteq C$.