MATH 3200 – Homework #1

posted August 19, 2024; due at the **start of class** on August 28, 2024

Assignments are expected to be **neat** and **stapled**. **Illegible work may not be marked.** You will be graded on the correctness of your answers *and* the coherence of your explanations.

1. Determine which of the following are statements. If not, explain why not. If so, determine the truth value of the statement.

   (a) Calvin Coolidge was the greatest American president.

   (b) The square root of a rational number is always a rational number.

   (c) When will the Philadelphia Phillies win the World Series?

   (d) This sentence is false.

   (e) $n(n+1)/2$.

2. Assign the following into groups of statements that have the same meaning.

   (a) $P \vee Q$,

   (b) P and Q,

   (c) at least one of the following is true: P, Q,

   (d) All of the following are true: P, Q,

   (e) Each of the following is true: P, Q.

3. Use truth tables to prove the following equivalences between compound statements involving P, Q, and R. Remember that $\equiv$ is our shorthand for "is logically equivalent to."

   (a) $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge (P \wedge R)$,

   (b) $P \vee (Q \vee R) \equiv (P \vee Q) \vee (P \vee R)$,

   (c) $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$.

4. Determine the truth or falsity of the following statements. (You may look up non-mathematical facts.)

   (a) If Homer Simpson is the President of the United States, then Marge Simpson is the Queen of England.

   (b) If Jupiter has more than three moons, then we are living in the 21st century.

   (c) If Earth is the center of the universe, then $3 = 5$.

5. Consider a deck of cards where each card has a letter on one side and a number on the other. Now suppose that you draw four cards so that the face-up sides show

   $$C - 3 - 8 - E.$$

   Determine which cards would need to be flipped over in order to verify the truth value of the statement "If one side of a card is a vowel, then the number on the other side is odd."

6. Use the method of truth tables to show that the following statements are logically equivalent:

$$\text{If } P\text{, then } Q \text{ or } R, \quad \text{i.e.,} \quad P \Rightarrow (Q \vee R)$$

and

$$P \text{ and not } Q\text{, implies } R, \quad \text{i.e.,} \quad (P \wedge \neg Q) \Rightarrow R.$$

7. Which of the following are true for all real numbers x and y? In explaining your answers, you may assume familiar rules of inequalities.

   (a) $0 < x < 1$ and $0 < y < 1$ implies $0 < x + y < 2$.

   (b) $0 < x < 1$ and $0 < x + y < 2$ implies $0 < y < 1$.

   Notice that the statements in (a) and (b) depend on the values chosen for x and y! Your job is to determine whether (a) is a true statement for all x and y and whether (b) is a true statement for all x and y.

8. (a) Create a truth table for the compound statement

$$((P \Rightarrow Q) \ \wedge \ P) \Rightarrow Q.$$

   For which truth values of P and Q does this statement hold?

   (b) Create a truth table for the compound statement

$$((P \Rightarrow Q) \ \wedge \ (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R).$$

   For which truth values of P, Q, R does this statement hold?