MATH 4000/6000: Modern Algebra, Spring 2025

pollack.uga.edu/4000s25.html

Instructor:
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Office hours: TBA!

Course synopsis

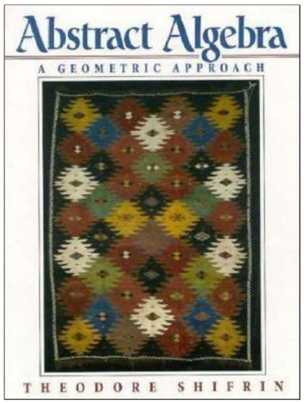
At this point in your mathematical career, you have accumulated a wealth of experience computing with integers, real numbers, and complex numbers. These items are so familiar to you that they may even have the appearance of being God-given --as if understanding those objects is What Mathematics is All About.

Abstract algebra challenges this notion. Integers, real numbers, and complex numbers are indeed fantastic, but they are fantastic not because they are handed down from on high, but because they have a rich and theory with useful consequences. And given how useful these objects are, we are compelled to isolate (and consider in the abstract) their most important properties. Once we do so, we find that there are many other objects with the same sorts of interesting properties. For example, we will see that the integers are an example of what is called a *ring*, that the real numbers and complex numbers are *fields*, and that the nonzero real numbers form a *group*. These more abstract-seeming objects are not just interesting in an intellectual let's-talk-about this-over-coffee kind of way, but understanding them deeply often leads to a new understanding of the objects of original interest.

Let me try to bring this back down to earth: Some primes, like 5, can be written as a sum of two squares: $5 = 2^2 + 1^2$. And other primes, like 3, cannot. The question of when this is possible is a question about the integers. But the easiest way to answer this question is to visit a totally different mathematically system, the *ring of Gaussian integers*. MATH 6000 students can expect problems about this!

(The official UGA course description, including learning outcomes, is attached.)

Textbook (required)



Abstract Algebra: A Geometric Approach by Theodore Shifrin

We will aim to cover Chapters 1--4 and parts of Chapter 6. For further adventures in Algebra-land, I encourage you to take MATH 4010/6010.

Homework/Exams/Grading

There will be three fifty-minute **in-class** exams, as well as a final exam.

- Midterm #1: Friday, February 14
- Midterm #2: Friday, March 28
- Midterm #3: Friday, April 18
- Final exam: Wednesday, April 30 12:00 - 3 PM (location TBA)

No make-up exams will be given. The final exam **is cumulative**. Your grade is made up of the following weighted components:

- Each midterm: 15% (total of 45%
- Homework: 25%
- Final exam: 30%

This class falls into the interactive lecture genre (not entirely unrelated to the practice of call and response in a liturgical context). What this means is that I intend to punctuate the lectures frequently with questions for you. For the show to go on, class participation is absolutely essential. Since you cannot participate in class if you are not present in class, your attendance is required. In particular, more than four unexcused absences may result in you being automagically withdrawn from the class. Of course, missing class is sometimes a necessity; keep me posted whenever you have a conflict and we should not have any issues.

Homework will be collected roughly once each week. As a general rule, late homework assignments are not accepted. Your lowest HW score will be dropped at the end of the semester.

All exams are closed book and closed notes.

Students enrolled in MATH 6000 will take the same exams as the students in MATH 4000 but will be assigned additional homework problems.

You are not only allowed, but **encouraged** to collaborate with your classmates on the homework assignments. The joy of mathematical discovery was meant to be shared! Having said that, collaboration does not mean copying, and consulting AI tools such as ChatGPT is not "collaboration" in the sense intended. You may not copy solutions from a textbook, classmate, website (including an AI tool), etcetera, and you must be the one to handwrite (or type) your solutions.

By entering UGA, you have already agreed to abide by the UGA honor code: "I will be academically honest in all of my academic work and will not tolerate academic dishonesty of others." A Culture of Honesty, the University's policy and procedures for handling cases of suspected dishonesty, can be found at honesty.uga.edu. A good rule of thumb is that you should be able to explain any work you turn in to a hypothetical interrogator.

Special accommodations

Students with disabilities who may require special accommodations should talk to me as soon as possible. Appropriate documentation concerning disabilities may be required. For further information, please visit the Disability Resource Center page.

UGA Well-being resources

UGA Well-being Resources promote student success by cultivating a culture that supports a more active, healthy, and engaged student community.

Anyone needing assistance is encouraged to contact Student Care & Outreach (SCO) in the Division of Student Affairs at 706-542-8479 or visit sco.uga.edu. Student Care & Outreach helps students navigate difficult circumstances by connecting them with the most appropriate resources or services. They also administer the Embark@UGA program which supports students experiencing, or who have experienced, homelessness, foster care, or housing insecurity.

UGA provides both clinical and non-clinical options to support student well-being and mental health, any time, any place. Whether on campus, or studying from home or abroad, UGA Well-being Resources are here to help.

Additional information, including free digital well-being resources, can be accessed through the UGA app or by visiting https://well-being.uga.edu.

FERPA Notice

The Federal Family Educational Rights and Privacy Act (FERPA) grants students certain information privacy rights. See the registrar's explanation at reg.uga.edu/general-information/ferpa/. FERPA allows disclosure of directory information (name, address, telephone, email, major, activities, degrees, awards, prior schools), unless requested in a written letter to the registrar.

Disclaimer

The course syllabus is a general plan for the course; deviations announced to the class by the instructor may be necessary.

APPROVED COURSE

COURSE ID

MATH 4000/6000

TITLES

Course Title: Modern Algebra I
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COURSE DESCRIPTION

An introduction to abstract algebra. Course begins with arithmetic and congruence in the integers, and introduces modular arithmetic. Course then moves to the more general setting of rings, and extends concepts from the integers to polynomial rings and other rings. The concepts of ideals and quotient rings are introduced. Groups are introduced, and normal subgroups and quotient groups appear as the analogues of ring-theoretic concepts.

ADDITIONAL REQUIREMENTS FOR GRADUATE STUDENTS

Extra problems on weekly homework.

GRADING SYSTEM

A-F (Traditional)

CREDIT HOURS AND LECTURE/LAB/DISCUSSION HOURS

	FIXED	VARIABLE
Credit Hours	3	
Lecture Hours per week	3	

NON-TRADITIONAL FORMAT (IF THE LECTURE/LAB HOURS OR THE LECTURE/DISCUSSION HOURS ARE FEWER THAN THE CREDIT HOURS, PLEASE PROVIDE JUSTIFICATION IN THE BOX BELOW.)

REPEAT POLICY

Course cannot be repeated for credit

EQUIVALENT COURSES

The course will not be open to students who have credit in the following courses:

Undergraduate

Graduate

REQUIRED PREREQUISITES

Undergraduate:

(MATH 3000 or MATH 3300 or MATH 3300E or MATH 3500 or MATH 3500H) and (MATH 3200 or MATH 3200W or CSCI 2610 or CSCI 2610E)

Graduate:

PREREQUISITE OR COREQUISITE COURSES

Undergraduate:

Graduate:

COREQUISITE COURSES

Undergraduate:

Graduate:

PRIMARY DELIVERY MECHANISM (SELECT ONLY ONE)

Lecture

COURSE WILL BE OFFERED

Every Year - Fall Spring Summer

EFFECTIVE SEMESTER AND YEAR OF CURRENT VERSION OF COURSE

Fall 2024

ADDITIONAL INFORMATION REQUIRED FOR THE SYLLABUS

COURSE OBJECTIVES OR EXPECTED LEARNING OUTCOMES

The Student Learning Outcomes are as follows:

- Students will acquire computational skills with modular arithmetic and polynomials, as well as with concrete example of rings and groups.
- Students will master basic definitions related to abstract algebraic structures such as rings, fields, groups, ideals, and quotients.
- Students will develop their abstract reasoning and proof-writing skills, enabling them to write rigorous proofs about rings and groups in both general and concrete settings.

TOPICAL OUTLINE

- Arithmetic in Z (the integers)
- Congruence in Z and modular arithmetic
- Rings
- Congruence in F[x] and congruence-class arithmetic
- Ideals and quotient rings
- Groups
- Normal subgroups and quotient groups

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UNIVERSITY HONOR CODE AND ACADEMIC HONESTY POLICY +

UGA WELL-BEING RESOURCES +

Back to MATH course list