A TALE OF FOUR RINGS

The ring R: $(\mathbb{Z}/2\mathbb{Z})[x]/\langle x^3+x+1\rangle$

Addition Table

$+$	0	1	α	$\alpha+1$	α^2	α^2+1	$\alpha^2+\alpha$	$\alpha^2+\alpha+1$
0	0	1	α	$\alpha+1$	α^2	α^2+1	$\alpha^2+\alpha$	$\alpha^2+\alpha+1$
1	1	0	$\alpha+1$	α	α^2+1	α^2	$\alpha^2+\alpha+1$	$\alpha^2+\alpha$
α	α	$\alpha+1$	α^2	$\alpha^2+\alpha$	$\alpha+1$	1	$\alpha^2+\alpha+1$	α^2+1
$\alpha+1$	$\alpha+1$	α	$\alpha^2+\alpha$	α^2	$\alpha^2+\alpha+1$	α^2	1	$\alpha+1$
α^2	α^2	α^2+1	$\alpha+1$	$\alpha^2+\alpha+1$	$\alpha^2+\alpha$	α	α^2+1	1
α^2+1	α^2+1	α^2	1	α^2	$\alpha^2+\alpha+1$	$\alpha^2+\alpha$	α	$\alpha+1$
$\alpha^2+\alpha$	$\alpha^2+\alpha$	$\alpha^2+\alpha+1$	$\alpha^2+\alpha+1$	1	α^2+1	α	$\alpha+1$	α^2
$\alpha^2+\alpha+1$	$\alpha^2+\alpha+1$	$\alpha^2+\alpha$	α^2+1	$\alpha+1$	1	$\alpha^2+\alpha$	α^2	α

Multiplication Table

$\times$	0	1	α	$\alpha+1$	α^2	α^2+1	$\alpha^2+\alpha$	$\alpha^2+\alpha+1$
0	0	0	0	0	0	0	0	0
1	0	1	α	$\alpha+1$	α^2	α^2+1	$\alpha^2+\alpha$	$\alpha^2+\alpha+1$
α	0	α	α^2	$\alpha^2+\alpha$	$\alpha+1$	1	$\alpha^2+\alpha+1$	α^2+1
$\alpha+1$	0	$\alpha+1$	$\alpha^2+\alpha$	α^2+1	$\alpha^2+\alpha+1$	α^2	1	α
α^2	0	α^2	$\alpha+1$	$\alpha^2+\alpha+1$	$\alpha^2+\alpha$	α	α^2+1	1
α^2+1	0	α^2+1	1	α^2	$\alpha^2+\alpha+1$	$\alpha^2+\alpha$	α	$\alpha+1$
$\alpha^2+\alpha$	0	$\alpha^2+\alpha$	$\alpha^2+\alpha+1$	1	α^2+1	α	$\alpha+1$	α^2
$\alpha^2+\alpha+1$	0	$\alpha^2+\alpha+1$	α^2+1	α	1	$\alpha^2+\alpha$	α^2	$\alpha+1$

The ring S: $(\mathbb{Z}/2\mathbb{Z})[x]/\langle x^3+x^2+1\rangle$

Addition Table

$+$	0	1	β	$\beta+1$	β^2	β^2+1	$\beta^2+\beta$	$\beta^2+\beta+1$
0	0	1	β	$\beta+1$	β^2	β^2+1	$\beta^2+\beta$	$\beta^2+\beta+1$
1	1	0	$\beta+1$	β	β^2+1	β^2	$\beta^2+\beta+1$	$\beta^2+\beta$
β	β	$\beta+1$	0	1	$\beta^2+\beta$	$\beta^2+\beta+1$	β^2	β^2+1
$\beta+1$	$\beta+1$	β	1	0	$\beta^2+\beta+1$	$\beta^2+\beta$	β^2+1	β^2
β^2	β^2	β^2+1	$\beta^2+\beta$	$\beta^2+\beta+1$	0	1	β	$\beta+1$
β^2+1	β^2+1	β^2	$\beta^2+\beta+1$	$\beta^2+\beta$	1	0	$\beta+1$	β
$\beta^2+\beta$	$\beta^2+\beta$	$\beta^2+\beta+1$	β^2	β^2+1	β	$\beta+1$	0	1
$\beta^2+\beta+1$	$\beta^2+\beta+1$	$\beta^2+\beta$	β^2+1	β^2	$\beta+1$	β	1	0

Multiplication Table

$\times$	0	1	β	$\beta+1$	β^2	β^2+1	$\beta^2+\beta$	$\beta^2+\beta+1$
0	0	0	0	0	0	0	0	0
1	0	1	β	$\beta+1$	β^2	β^2+1	$\beta^2+\beta$	$\beta^2+\beta+1$
β	0	β	β^2	$\beta^2+\beta$	β^2+1	$\beta^2+\beta+1$	1	$\beta+1$
$\beta+1$	0	$\beta+1$	$\beta^2+\beta$	β^2+1	1	β	$\beta^2+\beta+1$	β^2
β^2	0	β^2	β^2+1	1	$\beta^2+\beta+1$	$\beta+1$	β	$\beta^2+\beta$
β^2+1	0	β^2+1	$\beta^2+\beta+1$	β	$\beta+1$	$\beta^2+\beta$	β^2	1
$\beta^2+\beta$	0	$\beta^2+\beta$	1	$\beta^2+\beta+1$	β	β^2	$\beta+1$	β^2+1
$\beta^2+\beta+1$	0	$\beta^2+\beta+1$	$\beta+1$	β^2	$\beta^2+\beta$	1	β^2+1	β

The ring T: $(\mathbb{Z}/2\mathbb{Z})[x]/\langle x^3+x^2+x\rangle$

Addition Table

$+$	0	1	γ	$\gamma+1$	γ^2	γ^2+1	$\gamma^2+\gamma$	$\gamma^2+\gamma+1$
0	0	1	γ	$\gamma+1$	γ^2	γ^2+1	$\gamma^2+\gamma$	$\gamma^2+\gamma+1$
1	1	0	$\gamma+1$	γ	γ^2+1	γ^2	$\gamma^2+\gamma+1$	$\gamma^2+\gamma$
γ	γ	$\gamma+1$	0	1	$\gamma^2+\gamma$	$\gamma^2+\gamma+1$	γ^2	γ^2+1
$\gamma+1$	$\gamma+1$	γ	1	0	$\gamma^2+\gamma+1$	$\gamma^2+\gamma$	γ^2+1	γ^2
γ^2	γ^2	γ^2+1	$\gamma^2+\gamma$	$\gamma^2+\gamma+1$	0	1	γ	$\gamma+1$
γ^2+1	γ^2+1	γ^2	$\gamma^2+\gamma+1$	$\gamma^2+\gamma$	1	0	$\gamma+1$	γ
$\gamma^2+\gamma$	$\gamma^2+\gamma$	$\gamma^2+\gamma+1$	γ^2	γ^2+1	γ	$\gamma+1$	0	1
$\gamma^2+\gamma+1$	$\gamma^2+\gamma+1$	$\gamma^2+\gamma$	γ^2+1	γ^2	$\gamma+1$	γ	1	0

Multiplication Table

$\times$	0	1	γ	$\gamma+1$	γ^2	γ^2+1	$\gamma^2+\gamma$	$\gamma^2+\gamma+1$
0	0	0	0	0	0	0	0	0
1	0	1	γ	$\gamma+1$	γ^2	γ^2+1	$\gamma^2+\gamma$	$\gamma^2+\gamma+1$
γ	0	γ	γ^2	$\gamma^2+\gamma$	$\gamma^2+\gamma$	γ^2	γ	0
$\gamma+1$	0	$\gamma+1$	$\gamma^2+\gamma$	γ^2+1	γ	1	γ^2	$\gamma^2+\gamma+1$
γ^2	0	γ^2	$\gamma^2+\gamma$	γ	γ	$\gamma^2+\gamma$	γ^2	0
γ^2+1	0	γ^2+1	γ^2	1	$\gamma^2+\gamma$	$\gamma+1$	γ	$\gamma^2+\gamma+1$
$\gamma^2+\gamma$	0	$\gamma^2+\gamma$	γ	γ^2	γ^2	γ	$\gamma^2+\gamma$	0
$\gamma^2+\gamma+1$	0	$\gamma^2+\gamma+1$	0	$\gamma^2+\gamma+1$	0	$\gamma^2+\gamma+1$	0	$\gamma^2+\gamma+1$

The ring U: $(\mathbb{Z}/2\mathbb{Z})[x]/\langle x^3+x^2\rangle$

Addition Table

$+$	0	1	δ	$\delta+1$	δ^2	δ^2+1	$\delta^2+\delta$	$\delta^2+\delta+1$
0	0	1	δ	$\delta+1$	δ^2	δ^2+1	$\delta^2+\delta$	$\delta^2+\delta+1$
1	1	0	$\delta+1$	δ	δ^2+1	δ^2	$\delta^2+\delta+1$	$\delta^2+\delta$
δ	δ	$\delta+1$	0	1	$\delta^2+\delta$	$\delta^2+\delta+1$	δ^2	δ^2+1
$\delta+1$	$\delta+1$	δ	1	0	$\delta^2+\delta+1$	$\delta^2+\delta$	δ^2+1	δ^2
δ^2	δ^2	δ^2+1	$\delta^2+\delta$	$\delta^2+\delta+1$	0	1	δ	$\delta+1$
δ^2+1	δ^2+1	δ^2	$\delta^2+\delta+1$	$\delta^2+\delta$	1	0	$\delta+1$	δ
$\delta^2+\delta$	$\delta^2+\delta$	$\delta^2+\delta+1$	δ^2	δ^2+1	δ	$\delta+1$	0	1
$\delta^2+\delta+1$	$\delta^2+\delta+1$	$\delta^2+\delta$	δ^2+1	δ^2	$\delta+1$	δ	1	0

Multiplication Table

$\times$	0	1	δ	$\delta+1$	δ^2	δ^2+1	$\delta^2+\delta$	$\delta^2+\delta+1$
0	0	0	0	0	0	0	0	0
1	0	1	δ	$\delta+1$	δ^2	δ^2+1	$\delta^2+\delta$	$\delta^2+\delta+1$
δ	0	δ	δ^2	$\delta^2+\delta$	δ^2	$\delta^2+\delta$	0	δ
$\delta+1$	0	$\delta+1$	$\delta^2+\delta$	δ^2+1	0	$\delta+1$	$\delta^2+\delta$	1
δ^2	0	δ^2	δ^2	0	δ^2	0	0	δ^2
δ^2+1	0	δ^2+1	$\delta^2+\delta$	$\delta+1$	0	δ^2+1	$\delta^2+\delta$	$\delta+1$
$\delta^2+\delta$	0	$\delta^2+\delta$	0	$\delta^2+\delta$	0	$\delta^2+\delta$	0	$\delta^2+\delta$
$\delta^2+\delta+1$	0	$\delta^2+\delta+1$	δ	1	δ^2	$\delta+1$	$\delta^2+\delta$	1