Bibliography

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The following is an (incomplete!) list of written resources relevant to the lectures. As with any list of this type, there is a trail in both directions — papers cited and papers that cite these — that you are encouraged to explore.

References

[1] L. Carlitz. Some theorems on irreducible reciprocal polynomials over a finite field. *J. Reine Angew. Math.*, 227:212–220, 1967.

> Original proof of the formula for the number of self-reciprocal irreducibles of a given degree.

[2] K. Conrad. Expository papers. Website at `http://www.math.uconn.edu/~kconrad/blurbs/`.

> Several expository notes spanning all areas of mathematics. Most relevant to our course are the notes on "analogies between $\mathbf{Z}$ and $\mathbf{F}[T]$", "quadratic reciprocity in odd characteristic", and "quadratic reciprocity in characteristic 2".

[3] K. Conrad. Irreducible values of polynomials: a non-analogy. In *Number fields and function fields—two parallel worlds*, volume 239 of *Progr. Math.*, pages 71–85. Birkhäuser Boston, Boston, MA, 2005.

> Expository paper describing a surprising **non**-analogy between integers and polynomials. In particular, it is shown that for certain irreducible polynomials $f(t, x) \in \mathbf{F}_q[t, x]$, there are **no** polynomials $g(t) \in \mathbf{F}_q[t]$ with $f(t, g(t))$ irreducible, despite the lack of local obstructions.

[4] G. Effinger. Toward a complete twin primes theorem for polynomials over finite fields. In *Finite fields and applications*, volume 461 of *Contemp. Math.*, pages 103–110. Amer. Math. Soc., Providence, RI, 2008.

Contains a proof of the twin prime conjecture for polynomials in the case $q = 3$. Also contains some observations concerning the still unresolved case $q = 2$.

[5] G. Frei. The unpublished section eight: on the way to function fields over a finite field. In *The shaping of arithmetic after C. F. Gauss's Disquisitiones arithmeticae*, pages 159–198. Springer, Berlin, 2007.

Historical survey detailing Gauss's work on the theory of finite fields and polynomials over a finite field.

[6] A. Granville. Analytic number theory. In *Princeton companion to mathematics*. Princeton University Press, 2008.

Expository article on analytic number theory; contains accessible presentations of many of the key ideas in prime number theory.

[7] A. Granville and T. J. Tucker. It's as easy as *abc*. *Notices Amer. Math. Soc.*, 49(10):1224–1231, 2002.

Expository article concerning Fermat's last theorem for polynomials, Mason's theorem, and the *abc* conjecture.

[8] C. Hall. L-functions of twisted Legendre curves. *J. Number Theory*, 119(1):128–147, 2006.

Hall's theorem on the polynomial twin prime conjecture is proved here. (But as the title suggests, this isn't the focus of the paper.)

[9] J.-R. Joly. Sommes des carrés dans certains anneaux principaux. *Bull. Sci. Math. (2)*, 94:85–95, 1970.

This article contains (among other things) a two-squares theorem for $F[i]$ for not-necessarily-finite fields F; this generalizes a result of Leahey.

[10] W. Leahey. Sums of squares of polynomials with coefficients in a finite field. *Amer. Math. Monthly*, 74:816–819, 1967.

A proof of the two-squares theorem for $\mathbf{F}_q[t]$.

[11] Y.-R. Liu and T. D. Wooley. The unrestricted variant of Waring's problem in function fields. *Funct. Approx. Comment. Math.*, 37(part 2):285–291, 2007.

Recent paper describing the best known upper bounds on $w(k, \mathbf{F}_q[t])$.

[12] H. Meyn and W. Götz. Self-reciprocal polynomials over finite fields. In *Séminaire Lotharingien de Combinatoire (Oberfranken, 1990)*, volume 413 of *Publ. Inst. Rech. Math. Av.*, pages 82–90. Univ. Louis Pasteur, Strasbourg, 1990.

Contains the simple proof we gave of Carlitz's formula for the number of self-reciprocal irreducibles of a given degree.

[13] M. B. Nathanson. *Additive number theory: the classical bases*, volume 164 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1996.

If you want to see (a streamlined version of) Hilbert's solution to Waring's problem, look here.

[14] R. Paley. Theorems on polynomials in a Galois field. *Quarterly J. Math.*, 4:52–63, 1933.

The first proof that $w(k, \mathbf{F}_q[t]) < \infty$ for all k and q.

[15] P. Pollack. An explicit approach to Hypothesis H for polynomials over a finite field. In *Anatomy of integers*, volume 46 of *CRM Proc. Lecture Notes*, pages 259–273. Amer. Math. Soc., Providence, RI, 2008.

This paper contains a proof of the twin prime conjecture for polynomials in the case $q = 3$. Also here is a proof that there are infinitely many irreducibles of the form $f^2 + 1$ over $\mathbf{F}_q$ in the case $q \equiv 3 \bmod 4$. Several generalizations are discussed.

[16] P. Pollack. Simultaneous prime specializations of polynomials over finite fields. *Proc. Lond. Math. Soc. (3)*, 97(3):545–567, 2008.

Among other things, this paper establishes the expected asymptotic formulas for the number of twin prime pairs of a given degree n over $\mathbf{F}_q$, and for the number of irreducibles of the form $f^2 + 1$ with f of a given degree n, in the case when n is fixed and $q \to \infty$. Unfortunately, the results are incomplete, in that extra restrictions on the characteristic of $\mathbf{F}_q$ have to be imposed for the proofs to work. These restrictions were mostly removed in later work of L. Bary-Soroker; see his impressive paper in *Adv. Math.* 229:854–874, 2012.

[17] P. Pollack. Revisiting Gauss's analogue of the prime number theorem for polynomials over a finite field. *Finite Fields Appl.*, 16(4):290–299, 2010.

Examines in detail (more detail than in our lectures) the analogy between the classical prime number theorem and Gauss's polynomial prime number theorem.

[18] P. Roquette. Manuscripts. Website at `http://www.rzuser.uni-heidelberg.de/~ci3/manu.html`.

Many valuable historical manuscripts; of particular relevance is the series of papers titled "The Riemann hypothesis in charateristic p, its origin and development".

[19] M. Rosen. *Number theory in function fields*, volume 210 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 2002.

Textbook whose first few chapters give a very down-to-earth treatment of analogies between $\mathbf{Z}$ and $\mathbf{F}_q[t]$. This was the primary source for our proof of the quadratic reciprocity law in $\mathbf{F}_q[t]$.

[20] N. Snyder. An alternate proof of Mason's theorem. *Elem. Math.*, 55(3):93–94, 2000.

The source for our simple proof of Mason's theorem.

[21] L. Tornheim. Sums of n-th powers in fields of prime characteristic. *Duke Math. J.*, 4:359–362, 1938.

This paper proves that if F is a field of positive characteristic, then $\Sigma(k, F)$ is also a field, and that if F is finite, then $w(k, F) \leq k$.

[22] L. N. Vaserstein. Waring's problem for algebras over fields. *J. Number Theory*, 26(3):286–298, 1987.

Our proof of Paley's theorem (concerning Waring's problem in $\mathbf{F}_q[t]$) is based on the ideas explained here.

[23] L. N. Vaserstein. Waring's problem for commutative rings. *J. Number Theory*, 26(3):299–307, 1987.

It was claimed in the lectures that in every ring R (assumed commutative, with 1), every element that is a finite sum of 11th powers is a sum of at most 11^3 such powers. The proof is here.

[24] D. Wan. Generators and irreducible polynomials over finite fields. *Math. Comp.*, 66(219):1195–1212, 1997.

Carefully written research paper. So carefully and clearly written that it also serves as a useful (though not comprehensive) reference for character sum estimates that follow from Weil's analogue of the Riemann Hypothesis.

[25] E. M. Wright. An easier Waring's problem. *J. London Math. Soc.*, 9:267–272, 1934.

This paper contains a simple proof that for every odd k, all integers are the sum at most $w(k, \mathbf{Z}) < \infty$ kth powers. Moreover, the same holds for even exponents k provided that one allows the summands to be taken with **either sign**.