ERRATA TO "DISTRIBUTION IN COPRIME RESIDUE CLASSES OF POLYNOMIALLY-DEFINED MULTIPLICATIVE FUNCTIONS"

(1) In the paragraph following the statement of Theorem 1.3, it is claimed that $D^{\omega(q)} > (\log x)^{(1+\delta)\alpha(q)}$ can happen already with $\log q$ of order $\log_2 x/(\log_3 x)^{D-1}$. What should have been claimed is that this can happen for $\log q \ll_D \log_2 x$ (this weaker result is all that is needed to show condition (i) reflects a genuine obstruction to uniformity). It suffices to take q as the product of primes from $D+1$ to $K_D \log x$, where K_D is a large constant depending on D. (To estimate $\alpha(q)$ we use that F has on average one root per prime, which is a consequence of the prime ideal theorem applied to the number field cut out by F.)

(2) The argument for the absolute irreducibility of $F(x)F(y) - w$ appearing at the end of §6 requires repair. A correct proof is as follows:

Suppose that $F(x)F(y) - w = U(x,y)V(x,y)$ for some $U(x,y), V(x,y) \in \overline{\mathbb{F}}_\ell[x,y]$. Then for each root $\theta \in \overline{\mathbb{F}}_\ell$ of F, we find that $-w = U(\theta,y)V(\theta,y)$, and so in particular $U(\theta,y)$ is constant. Thus, if we write

$$U(x,y) = \sum_{k\geq 0} a_k(x) y^k,$$

with each $a_k(x) \in \overline{\mathbb{F}}_\ell[x]$, then $a_k(\theta) = 0$ for each $k > 0$. Since F has no multiple roots over $\overline{\mathbb{F}}_\ell$, each such $a_k(x)$ is forced to be a multiple of $F(x)$, hence $U(x,y) \equiv a_0(x) \pmod{F(x)}$. A symmetric argument shows that $V(x,y) \equiv b_0(y) \pmod{F(y)}$ for some $b_0(y) \in \overline{\mathbb{F}}_\ell[y]$, so that $V(x,\theta) = b_0(\theta)$. Consequently, for any root $\theta \in \overline{\mathbb{F}}_\ell$ of F,

$$-w \equiv F(x)F(\theta) - w \equiv U(x,\theta)V(x,\theta) \equiv a_0(x)b_0(\theta) \pmod{F(x)},$$

which shows that $U(x,y) \equiv a_0(x) \equiv c \pmod{F(x)}$ for some constant $c \in \overline{\mathbb{F}}_\ell$. But this forces $c = U(\theta,\theta)$, showing that $F(x)$ divides $U(x,y) - U(\theta,\theta)$. By symmetry, so does $F(y)$, and we obtain $U(x,y) = U(\theta,\theta) + F(x)F(y)Q(x,y)$ for some $Q(x,y) \in \overline{\mathbb{F}}_\ell[x,y]$.[1] Degree considerations now imply that for $U(x,y)$ to divide $F(x)F(y) - w$, either $Q(x,y)$ is a nonzero constant, in which case $V(x,y)$ is constant, or $Q(x,y) = 0$, in which case $U(x,y)$ is constant.

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[1] In the published version, it was argued (incorrectly) that $F(x), F(y)$ divide $U(x,y) - U(0,0)$.